Computable categoricity relative to a c.e. degree

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Outline

Preliminaries

Definitions

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A structure $\mathcal{A}$ is **computably categorical** if for every computable copy $\mathcal{B}$ of $\mathcal{A}$, there exists a computable isomorphism between $\mathcal{A}$ and $\mathcal{B}$.

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- Computable ordered groups of finite rank (Gončarov, Lempp, Solomon [GLS03]).

The given conditions in each example are both necessary and sufficient for computable categoricity.

Relativizing categoricity

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Remark

If a structure is relatively computably categorical, then it is computably categorical.

The converse is not true in general.

Algebraic characterization of computable categoricity

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Theorem (Ash, Knight, Manasse, and Slaman [Ash+89]; Chisholm [Chi90])

A structure is relatively computably categorical if and only if it has a formally Σ_1 Scott family.

Scott families

Definition

A **Scott family of ∃-formulas** for a structure $\mathcal{A}$ is a set S of existential formulas such that

(1) for every $\overline{a} \in \mathcal{A}$, there is a $\varphi(\overline{x}) \in S$ such that $\mathcal{A} \models \varphi(\overline{a})$, and

(2) if $\mathcal{A} \models (\varphi(\overline{a}) \wedge \varphi(\overline{b}))$ for $\overline{a}, \overline{b} \in \mathcal{A}$ and $\varphi(\overline{x}) \in S$, then there is an automorphism of $\mathcal{A}$ sending $\overline{a}$ to $\overline{b}$.

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A **Scott family of $\exists$-formulas** for a structure $\mathcal{A}$ is a set S of existential formulas such that

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Observation

If a computable structure $\mathcal{A}$ has a Scott family of $\exists$-formulas, then $\mathbf{0}''$ can enumerate such a family. So, $\mathcal{A}$ has a formally Σ_1 Scott family relative to $\mathbf{0}''$.

Plain but not relative

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If a structure is computably categorical and its $\forall\exists$ *theory is decidable, then it is relatively computably categorical.*

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Kudinov [Kud96] showed that the assumption of 2-decidability could not be lowered to 1-decidability.

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A structure $\mathcal{A}$ is **relatively Δ^0_α-categorical** if for any copy $\mathcal{B}$ of $\mathcal{A}$, there is a $\Delta^0_\alpha(\mathcal{B})$-computable isomorphism between $\mathcal{A}$ and $\mathcal{B}$.

Plain and relative Δ^0_α-categoricity

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In the same paper, he showed that a computable Boolean algebra is *relatively* Δ^0_2-categorical if it is a finite direct sum of atoms, 1-atoms, and atomless elements.

Categoricity relative to a degree

A newer relativization

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For $X \in 2^{\mathbb{N}}$, a structure $\mathcal{A}$ is **computably categorical relative to a degree X** if for every X-computable copy $\mathcal{B}$ of $\mathcal{A}$, there is an X-computable isomorphism between $\mathcal{A}$ and $\mathcal{B}$.

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Fact

A computable structure $\mathcal{A}$ is **relatively computably categorical** *if for all $X \in 2^{\mathbb{N}}$, $\mathcal{A}$ is computably categorical relative to X.*

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Fact (Downey, Harrison-Trainor, Melnikov [DHTM21])

If $\mathcal{A}$ is a computable structure and it is computably categorical relative to some degree $\mathbf{d} \geq \mathbf{0}''$, then $\mathcal{A}$ has a $\mathbf{0}''$-computable Σ^0_1 Scott family. So, $\mathcal{A}$ is computably categorical relative to all $\mathbf{d} \geq \mathbf{0}''$.

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Proof.

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So at $\mathbf{0}''$ and above, any computable structure $\mathcal{A}$ will settle on whether it is computably categorical relative to all degrees or to none of them.

Question

What happens between $\mathbf{0}$ and $\mathbf{0}''$?

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Theorem (Downey, Harrison-Trainor, Melnikov [DHTM21])

There is a computable structure $\mathcal{A}$ and c.e. degrees $\mathbf{0} = Y_0 <_T X_0 <_T Y_1 <_T X_1 <_T \ldots$ such that

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(2) *$\mathcal{A}$ is not computably categorical relative to X_i for each i,*

(3) *$\mathcal{A}$ is computably categorical relative to $\mathbf{0}'$.*

Partial orders of c.e. degrees

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Theorem (V. [Vil24])

Let $P = (P, \leq)$ be a computable partially ordered set and let $P = P_0 \sqcup P_1$ be a computable partition. Then, there exists a computable computably categorical directed graph $\mathcal{G}$ and an embedding h of P into the c.e. degrees where $\mathcal{G}$ is computably categorical relative to each degree in $h(P_0)$ and is not computably categorical relative to each degree in $h(P_1)$.

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(1) embedding P into the c.e. degrees;
(2) making the graph $\mathcal{G}$ computably categorical;
(3) making $\mathcal{G}$ computably categorical relative to all degrees in $h(P_0)$; and
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(4) making $\mathcal{G}$ not computably categorical relative to any degree in $h(P_1)$.

We use a tree of strategies to organize restraints and parameters.

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Theorem (V. [Vil24])

There exists a computable computably categorical directed graph $\mathcal{G}$ and c.e. sets X_0 and X_1 such that

(1) *X_0 and X_1 form a minimal pair,*

(2) *$\mathcal{G}$ is not computably categorical relative to X_0 and to X_1, and*

(3) *$\mathcal{G}$ is computably categorical relative to $X_0 \oplus X_1$.*

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You can also form a minimal pair X_0 and X_1 where $\mathcal{G}$ is computably categorical relative to X_0 but not to X_1, and is computably categorical relative to $X_0 \oplus X_1$.

Future directions: given a c.e. degree

Another question you can ask is the following.

Question

Given an arbitrary c.e. set D, can you always build a computable graph $\mathcal{G}$ where

(1) *$\mathcal{G}$ is computably categorical, and*

(2) *$\mathcal{G}$ is not computably categorical relative to D?*

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Conjecture

Given an arbitrary c.e. set D, there is a computable graph $\mathcal{G}$ which is computably categorical and not computably categorical relative to D, and vice-versa.

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Thanks

Thank you for your attention!

I'd be happy to answer any questions.